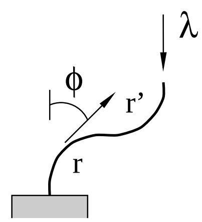
DNA Modeling Course
Exercise Session 8
Summer 2006 Part 1

Before proceeding to the following exercises make sure you have completed and understood Session 7, i.e. take your time to finish the previous Session. In particular use the General Auto Solution (point (d) of last Session) to visualize the solution, noting that with different choices of the Coordinate Axis you can plot r_1 and r_3 as a function of s but also ϕ as a function of s.

Problem 1

Consider, as in class, a planar, inextensible, unshearable elastic rod whose admissible configurations are described by a continuously differentiable angle function $\phi : [0,1] \to \mathbb{R}$ satisfying the imposed boundary conditions $\phi(0) = 0$. Let $\boldsymbol{r}(s) = (r_1(s), r_3(s))$ be the rod centerline parametrized by the arc length coordinate $s \in [0,1]$, $\lambda > 0$ a parameter representing an external downward force, and finally $K = 0.5 > 0$ a material constant. The reference strain is zero, i.e. $\hat{u}_2(s) \equiv 0$.
Consider the following boundary conditions with no lateral force ($\nu = 0$)

$$\phi(0) = 0, \quad r_1(0) = 0, \quad r_3(0) = 0,$$
$$N_3(1) = -\lambda, \quad \phi(1) = 0, \quad N_1(1) = 0.$$



(a) (trivial equilibrium) Write down the appropriate equilibrium equations for this problem and verify that $\phi_0(s) \equiv 0$ is an equilibrium for any $\lambda > 0$.

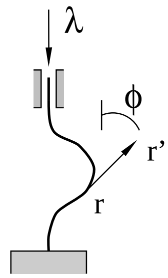
(b) (bifurcation points) At what values of λ can bifurcation from the trivial equilibrium ϕ_0 occur? (Solve a linearized system as done in class, for which λ does there exist a non-trivial solution closeby?)

(c) Use VBM (see instructions at the end) to get a bifurcation diagram and pictures of the rod. There are different sensible choices for the coordinate axis in the bifurcation diagram, e.g. consider the $(\lambda, m_2(1), \kappa)$- projection.

Problem 2

Consider Problem 1 again. Use the boundary condition $r_1(1) = 0$ instead of $N_1(1) = 0$, that is

$$\phi(0) = 0, \quad r_1(0) = 0, \quad r_3(0) = 0,$$
$$N_3(1) = -\lambda, \quad \phi(1) = 0 \quad r_1(1) = 0, .$$



(a) (trivial equilibrium) Write down the appropriate equilibrium equations for this problem and verify that $\phi_0(s) \equiv 0$ is an equilibrium for any $\lambda > 0$.

(b) (bifurcation points) At what values of λ can bifurcation from the trivial equilibrium ϕ_0 occur? Is this case the same as in Problem 1?

Show that there are two types of bifurcation points - a sequence with the horizontal end force $N_1(1) = \nu = 0$ interlaced with a sequence with the horizontal end force $\nu \neq 0$.

(c) Use VBM to get a bifurcation diagram and pictures of the rod.

VBM

- In order to run VBM remember to type: source ~jwalter/vbm_env.csh
  Create two folders in your folder VBM, typing:
  cp -r planar_strut planar_strut_problem1
  cp -r planar_strut planar_strut_problem2
  This commande will copy the folder planar_strut and name it planar_strut_problem1 or planar_strut_problem2.
- In each folder (i.e. for each problem) you have to change the boundary conditions declared in the strut_ivp.f file. Read the documentation in the strut_ivp.f file. To do so just use your favourite editor (like nedit or emacs).
- As usual, to start VBM, you have to be in the appropriate folder and type:
  VBM.py -f strut.vbm

Don't forget to press ENTER or DRAW anytime you change a value. VBM ignores it otherwise!

Some Unix Commands:

ls	lists all objects in the current folder
cd dir1	change current directory to dir1
cd ..	go to the upper directory
cp file1 file2	copy the file file1 and name the copy file2
emacs file1	opens the file file1 in the editor 'emacs'