DNA Modelling Course
Exercise Session 7
Summer 2006 Part 1

SOLUTIONS

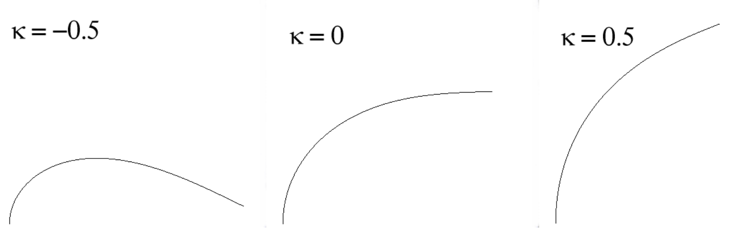
(a) Immediate.

(b)

	$\kappa = -0.5$	$\kappa = 0$	$\kappa = 0.5$
first bifurcation	$\lambda = 0.91729$	$\lambda = 1.23370$	$\lambda = 1.42530$
second bifurcation	$\lambda = 10.1265$	$\lambda = 11.1033$	$\lambda = 10.6601$

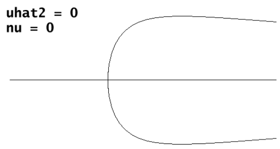
(c) The buckling load increases as κ varies from -0.5 to 0 to 0.5. This can be understood physically by the following. Suppose the $\kappa = -0.5$ and the $\kappa = 0$ rods are slightly buckled in such a way that they have the same small horizontal displacement at the top end, i.e. $r_1(1) = \epsilon$, $0 < \epsilon \ll 1$, for both rods. We want to compare the vertical loads λ responsible for these deformations. We first note that it is the curvature in the lower portion of the rod (close to $s = 0$) that is mainly responsible for the vertical displacement of the top end of the rod (curvature close to $s = 1$ does not affect $r_1(1)$ much). Second, the amplitude of the torque $m_2(s)$ is the product of the load λ by the moment arm $|r_1(1) - r_1(s)|$ and is therefore larger in the lower portion of the rod. We conclude that the essential of the bending occurs in the lower portion of the rod. Compared to the $\kappa = 0$ rod, the $\kappa = -0.5$ rod has a lower bending stiffness $K_2(s)$ in its lower portion, and therefore needs a smaller torque to satisfy the constraint $r_1(1) = \epsilon$. Since the torque is proportional to the load, the $\kappa = -0.5$ rod needs a smaller load to start to buckle, i.e. it has a smaller buckling load than the $\kappa = 0$ rod. A similar argument shows that the buckling load for the $\kappa = 0.5$ rod is higher than the buckling load for the $\kappa = 0$ rod.

(d) Here are the shapes of the rod in the $(\boldsymbol{e}_1, \boldsymbol{e}_3)$ plane for $\lambda = 1.7$, $\nu = 0$, $\hat{u}_2 = 0$ and for different values of κ:

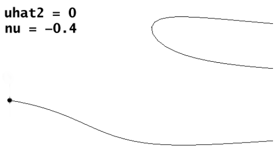


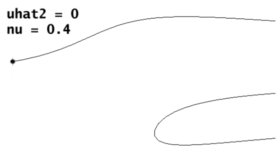
It is seen that under the same load, rods with lower values of κ buckle more.

(e) For $\hat{u}_2 = 0$ and $\nu = 0$ the boundary value problem (1)–(13) has the discrete symmetry that if $r_1(s)$ is a solution then $-r_1(s)$ is also a solution. The $[\lambda, r_1(1)]$ bifurcation diagram is therefore symmetric with respect to the $r_1(1) = 0$ axis. The portion shown below has a pitchfork bifurcation point (where the two branches intersect and one of them is vertical).

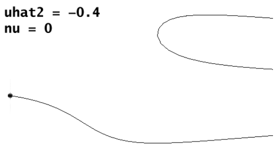


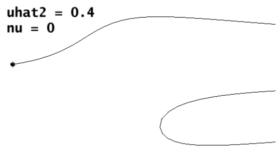
For $\hat{u}_2 = 0$ and $\nu \neq 0$ the symmetry is broken and the portion of the bifurcation diagram shown below has two disconnected branches. It has a fold point (where a branch is vertical).



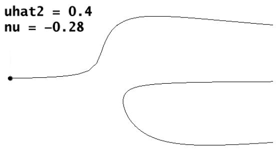


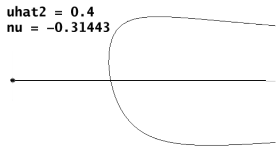
For $\hat{u}_2 \neq 0$ and $\nu = 0$ the bifurcation diagram is qualitatively (but not quantitatively) the same as for $\hat{u}_2 = 0$ and $\nu \neq 0$.

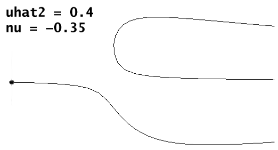




For fixed $\hat{u}_2 = 0.4$ the bifurcation diagram switches from “up” to “down” when ν goes through the value -0.31443. At the transition the bifurcation diagram is connected but not symmetric. It does not have a pitchfork bifurcation point but has a transcritical bifurcation point (where the two branches intersect) and a fold point (where one of the branches is vertical, above the transcritical bifurcation point).






uhat2 = 0.4
nu = −0.35